MCS CONTENT STANDARDS FOR KINDERGARTEN MATHEMATICS

Fluency Expectations or Examples of Culminating Standards

- K.OA.5: ***Fluently*** add and subtract within 5.

The following Standards have changes from the 2015-16 MS College- and Career-Readiness Standards:

Significant Changes (ex: change in expectations, new Standards, or removed Standards)
K.CC.7
K.OA.2
K.G.5

Slight Changes (slight change or clarification in wording)
K.OA.1
K.NBT.1

Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades K-5 Standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***fluently***). With respect to student performance and effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend to one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word ***fluently*** appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency is this sense that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

2016 Mississippi College- and Career-Readiness Standards for Mathematics, p. 19

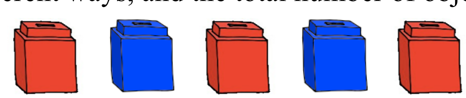
Counting and Cardinality

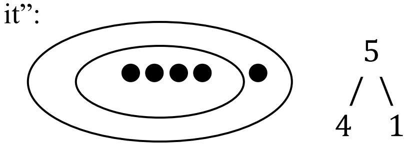
Cluster

Know number names and the count sequence.

Vocabulary: ones, tens, hundred

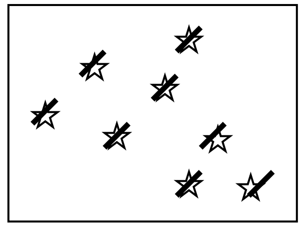
Standard	Clarifications
K.CC.1 Count to 100 by ones and by tens.	The focus of this Standard is on the counting number sequence, not on recognizing written numbers. "Learning a list of number names up to 100 is a challenging task for young children... The structure of the number names in a language is a major influence on the difficulties children have in learning to count correctly." (National Research Council, 2001) • In the English language, our words for "eleven" and "twelve" are based on old Anglo-Saxon words related to the idea of "one left" and "two left." We have inherited these words, but they do not fit the "pattern" of the rest of our number naming system very well. • The "phonetics" of our teen numbers in the English language can be confusing for students. – For example, "ten" becomes "teen," and "three" becomes "thir-." – We pronounce our "teen" numbers by naming the "smaller" value first: We write "thirteen" as 13. – We see "three" as "thir-" again in "thirty-one, thirty-two, thirty-three…" but then we write the "*first sound"* first: 31, 32, 33… It is normal for students to struggle with learning the sequence of number names, especially from 1 – 20, but our system of writing and naming numbers is simply a "convention" that we can help our students learn.
K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	Counting forward is more challenging for students than counting from 1 because they must pick up "in the middle" (so to speak) of the counting sequence and recognize what comes next. This requires a deeper understanding of our number system. Counting forward lays the foundation for students to "count on" to solve word problems in Grade 1 (**1.OA.6**), which is a more sophisticated (and efficient) strategy than "counting all" (starting at 1). Number lines and hundreds charts can help students develop familiarity with how our number system is organized. Over time, teachers can facilitate counting tasks by having students choose a number, close their eyes, picture the number line/hundreds chart in their head, and then think about "What comes next? And then what number?" This Standard is related to **K.CC.4c** in which students recognize that each number represents "one more" than the number before it.

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 – 20 (with 0 representing a count of no objects).	The goal of this Standard is for students to form a connection between a physical amount and the symbol (number) we use to represent "how many" that amount is. • Ex: Students may draw or count out nine objects and then write the number "9" to describe "how many." • Ex: Students may be given a number and asked to show "how many" that is with objects or pictures. ***<u>TEACHER NOTE</u>***: Due to varied development of fine motor and visual development, reversal of numerals is anticipated. While reversals should be pointed out to students and correct formation modeled in instruction, the emphasis of this standard is on the use of numerals to represent quantities rather than the correct handwriting formation of the actual numeral itself.
Cluster	
Count to tell the number of objects.	
Vocabulary: ones, tens, hundred	
K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. (*continued on next page*)	This Standard is based on research from the book *Adding It Up: Helping Children Learn Mathematics**. Students are not expected to know the terms (ex: cardinality) by name, but teachers should look for evidence of these understandings when working with students in counting. <u>5 Principles/Ideas that Students Need to Understand in Order to Count Meaningfully</u>: 1. **<u>Stable Order</u>** – Our counting numbers have a stable, reproducible sequence (**K.CC.4a**). -- Students who do not understand this may "invent" number names, such as "three-teen" or "ninety-twelve" or count out of sequence, "One, five, three,…" 2. **<u>One-to-One Correspondence</u>** – One number name/word goes with one object (**K.CC.4a**). -- Students who do not understand this may miss an item while counting a group, may "double" count an item while counting a group, or may simply repeat a counting sequence with no connection to the number of objects actually there. 3. **<u>Cardinality</u>** – The last number name said represents the total amount in the group (**K.CC.4b**). -- Suppose a student counts a group of ten items. If the teacher asks the student, "How many are there?" and he/she has to recount and/or "asks" (ex: "Ummm… ten?") instead of "tells" (ex: "Ten!"), then he/she does not have a strong understanding of cardinality. 4. **<u>Order Irrelevance</u>** – We can count a group of objects in different ways, and the total number of objects will not change (**K.CC.4b**). -- It is important for students to investigate this first-hand. Ex: Suppose we have the following group of snap cubes:  We can count the cubes in different ways (left to right, right to left, red first and then blue), but there will always be a total of 5 cubes. 5. **<u>Abstraction</u>** – We can count <u>anything</u> (ex: counting bears, snap cubes, cars, etc.) using our number system.

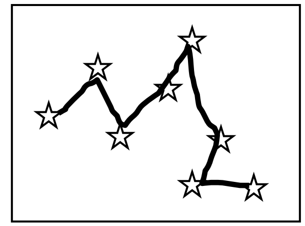
K.CC.4 (*cont'd*) Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	**K.CC.4c** is related to **K.CC.2**. The focus here is for students to recognize that as we "go forward" when counting, the amount increases by one each time. (Ex: "Five is one more than four.") Teachers can help to develop this understanding by asking students to act out and answer questions directly related to this concept throughout the year. <u>Example</u>: "There were four bears at the picnic." (Ask students to model the story using teddy bear counters or even act out the story themselves.) "If we had one more bear, how bears would there be?" (Students can then predict how many bears, model/act it out to see "how many," and then summarize, "Five is one more than four.") The ultimate goal is for students to understand that numbers are "nested" within larger numbers, although they are not expected to use that language. For example, "Five has four in it": This understanding will help them decompose numbers (**K.OA.3**), which lays the foundation for addition/subtraction strategies (**1.OA.6**).  * (National Research Council, 2001)
K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. (*continued on next page*)	Research has shown that there is a general progression of Kindergarteners' Counting Skills: 1. Say number names in correct sequence (**K.CC.1**). 2. Count objects in a line. -- This requires an understanding of one-to-one correspondence and cardinality (**K.CC.4**). Counting in a line tends to be easiest for students because that arrangement gives them a clear place to start and stop. 3. Count objects in a rectangular array. -- An array tends to challenge students more because they must find a way to count every row and column without skipping or repeating. -- Some students feel compelled to "start over" at 1 when they move from one row to another. They are still focusing on the individual "lines" (see #2 above) rather than seeing the rows as part of the same set. 4. Count objects in a circle. -- Because this arrangement "wraps around," students have to remember where they started and must stop at the object right before that one. -- This arrangement does offer the advantage of giving students a "path" to follow. They can start at the top and mark that object (with a finger or pencil mark) and then use a finger/pencil to "follow the path" around the circle.

K.CC.5 (*cont'd*)
Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects

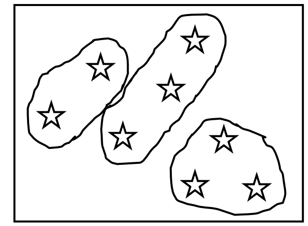
5. Count objects in a scattered arrangement. (*Notice that the number of objects is decreased to 10 here.)
-- This arrangement challenges students to use an organizational strategy to keep up with which objects they have already counted. (See examples below.) Some students may also see smaller groups within the larger group. This is part of *subitizing* (when students recognize "how many" without counting one at a time.)



"Marking off"



"Tracing a path"



"I saw it as 2, 3, and 3.
2 and 3 is 5, and 3 more is 8."

6. Count out a given number of objects.
-- This tends to be the most challenging because students must keep up with "how many" objects they are supposed to count out of a larger group, rather than just counting all of the objects until they reach the end.

Cluster

Compare numbers.

Vocabulary: compare, greater than, less than, equal to, more, fewer, tens, ones

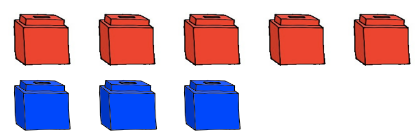
K.CC.6
Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]

[1] Include groups with up to ten objects.

(*continued on next page*)

Students may also use the terms "more than," "fewer than," or "the same as" as descriptive terms.

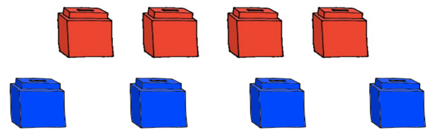
Initially students use a matching strategy to pair one object from each group and determine if one group has more or fewer objects than the other. In **K.CC.4**, "one-to-one correspondence" is used to describe matching one item to one number name. Another application of "one-to-one correspondence" is used here to pair an object from one group to an item from the other group.



As students progress in their understanding of the counting sequence and its relationship to "how many" our numbers represent, they may use this thinking to decide if one group contains more or less than another.

<u>Example</u>: "Which would be more? 5 cookies or 4 cookies?"

<u>Student</u>: "Five comes after four when you count, so five is more than four."

K.CC.6 (*cont'd*) Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] [1] Include groups with up to ten objects.	***TEACHER NOTE***: It is a natural misconception for young students to think that because one set of objects is "longer" than the other, it is "more than" the other. For example, many students will claim that there are "more" blue cubes than red cubes in the arrangement below because the row of blue cubes *looks* "bigger":  It is important for students to have multiple hands-on opportunities to pair, match, re-arrange, and count sets of objects in order to address this misunderstanding.
K.CC.7 Compare two numbers between 1 and 20 presented as written numerals.	This Standard builds off of students' work in **K.CC.3** and **K.CC.6**. Students need multiple experiences with actual sets of objects in order to develop a strong conceptual understanding of "how much/how many" certain numbers represent before they compare numbers written only as symbols. Students' ability to address "Which is more?" and "Which is less?" questions in Kindergarten lays the foundation for their in Grade 1 when they will model and solve "Compare" addition/subtraction situations (see Table 1) to answer questions such as, "How many more?" and "How many less?"

Operations and Algebraic Thinking

Cluster

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Vocabulary: join, add, separate, subtract, equal to, how many

K.OA.1
Represent addition and subtraction, in which all parts and whole of the problem are within 10, with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies whenever drawings are mentioned in the Standards.)

Before introducing symbols (+, -, =) and equations, Kindergarteners need numerous experiences to act out, directly model (with objects, fingers, etc.), or describe the action of *joining* or *taking away* groups of objects. The purpose of this Standard is to develop understanding of the concepts of addition and subtraction, more than reading/solving equations. Students will have more opportunities to work with equations in Grade 1.

At this stage, students are still developing the fine motor skills and language skills to articulate what they are doing. Their drawings may not be sophisticated, but they can still be descriptive and give us insight into how the students are thinking about the mathematics. (See examples in **K.OA.2**.)

Students at this age are often very literal and may need help in letting objects (ex: snap cubes, counting bears, etc.) represent items that are *not those things* in a story problem. Teachers can help students be more comfortable with this idea with suggestions such as, "Let's pretend the two-color counters are apples," or "Let's use tally marks to represent the people in the story."

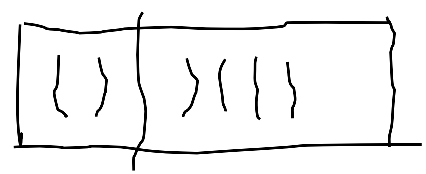
TEACHER NOTE: In Kindergarten, students may use fingers to represent parts of the problem (the amounts that are being joined, the total, the amount taken away, or the amount that is left). "Using fingers is not a concern [in Kindergarten] unless it remains at the first level of understanding in later grades."
(*Progressions for the CCSSM (Draft): K, Cardinality; K-5, Operations and Algebraic Thinking, May 2011, p. 4*)

K.OA.2
Solve addition and subtraction word problems within 10 involving situations of adding to, taking from, putting together and taking apart with unknowns in all positions by using objects or drawings to represent the problem.

(*continued on next page*)

Table 1 (at the end of this document) describes the types of addition/subtraction scenarios that Kindergarten students should explore (and "master") by the end of the year based on research supporting this progression:

Add to, Result Unknown	**Take from, Result Unknown**
Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?
Put Together/Take Apart, Total Unknown	**Put Together/Take Apart, Both Addends Unknown**
Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 4 + 1, 5 = 1 + 4, 5 = 2 + 3, 5 = 3 + 2, etc.

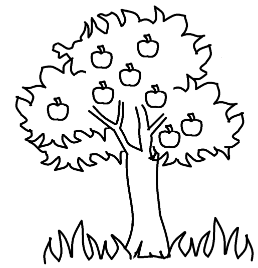
K.OA.2 (*cont'd*) Solve addition and subtraction word problems within 10 involving situations of adding to, taking from, putting together and taking apart with unknowns in all positions by using objects or drawings to represent the problem.	"Add to, Result Unknown" and "Take from, Result Unknown" tend to be the easiest problem types for children to solve because children can easily model/act out the action in the story, as it is presented. "Put Together/Take Apart" problem types tend to be more challenging for students because there is no "action" for them to do. ***TEACHER NOTE***: Because students' literacy skills are still in early development in Kindergarten, these story problems may be read aloud to students (multiple times, if needed) so that students can model/act out the story. This helps students to "mathematize" (focus on the mathematical relationships in) the story and to strengthen their active listening skills. Another option is to create story problems composed mostly (if not entirely) of sight words and use the problems to teach reading and mathematics concepts simultaneously. Example (Take from, Result Unknown): Nine grapes were in the bowl. I ate 3 grapes. How many grapes are in the bowl now? Student <talking>: I drew nine circles for grapes. I crossed out three 'cause you ate three. Then I counted how many were left. There were six grapes left.  Example **(**Put Together/Take Apart, Both Addends Unknown); Six crayons are in the box. Some are red, and some are blue. How many do you think could be red? How many do you think could be blue? Student <talking>: I drew a box. I drew two lines. Those are red crayons. Then I drew a line. Then I drew more lines and counted, "Three, four, five, six." Those are the blue ones. Four crayons are blue."  The teacher could then ask if there are *other* numbers of red/blue crayons that would fit the problem.
K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4+1).	This Standard is related to **K.CC.4c**. It is important for students to understand that numbers can be decomposed into smaller amounts in multiple ways. This understanding is fundamental to strategies for addition/subtraction they will use in Grade 1 (**1.OA.6**). (*continued on next page*)

K.OA.3 (*cont'd*)
Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4+1).

The book *Quack and Count* tells the story of 7 ducklings on an adventure. The pictures provide excellent opportunities to count the ducks in different arrangements. (Ex: How many ducks are in the water? How many ducks are not in the water?) The teacher can then record the number relationships (Ex: 7 = 3 + 4) on the board, as the class reads through the story.

Classroom tasks/discussions can also facilitate explorations of this concept throughout the year.

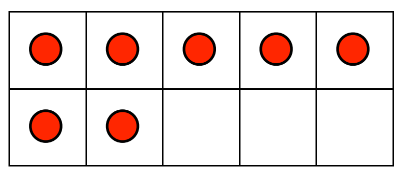
<u>Example</u>:
There are eight apples on the tree. Some are red, and some are yellow. Color how many apples you think are red and how many you think are yellow. Then we will share our pictures and work together to write equations that fit our pictures.



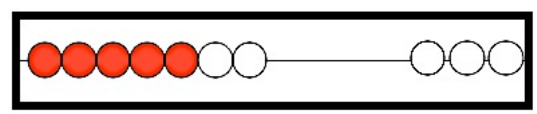
Kindergarteners need ample experiences breaking apart numbers and using vocabulary such as "and," "missing," and "the same amount as" before symbols (+, –, =) and equations (5 = 3 + 2) are introduced. Students should model the story with physical manipulatives or pictures *first*. If equations are used, they should then reflect/describe the student's model/picture.

"Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required." (*Mississippi College- and Career-readiness Standards, p. 25*)

Ten Frames and Rekenreks are powerful visual models to help students see the relationship between 5 and the larger numbers 6, 7, 8, and 9 (which can easily be thought of as "5 and some more"). For example, the number 7 can be described as "5 and 2 more" or "7 = 5 + 2."



Ten Frame



Rekenrek

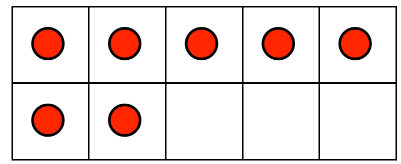
<u>TEACHER NOTE</u>: You may note that some equations have been written as "7 = 5 + 2," rather than the more familiar "5 + 2 = 7." It is important for students to have experience in seeing equations written both ways. The most common misconception of the equal sign is for students to interpret its meaning as "put the answer next," rather than as a statement of equality/balance. We can begin to prevent this misconception here.

K.OA.4
For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

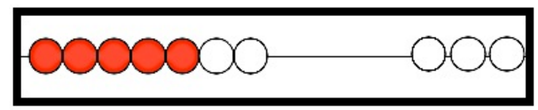
Ten Frames and Rekenreks (see **K.OA.3**) can be very powerful visual aids for this Standard, as well.

Example:
Farmer Ted packs ten apples in a crate. He has 7 apples. How many more apples does he need to fill a crate?

Student A <talking>: "I used counters. I put 7 apples in the crate. There are 3 spaces left. He needs three more apples."



Student B <talking>: "I used the Rekenrek. I pushed 7 beads over for the apples in the crate. There are three beads left. So, he needs three more apples."



As students work with these models throughout the year, they internalize the number relationships. Some may even visualize/describe numbers in terms of them.

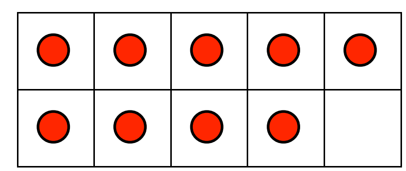
Example:
If I have 9 cupcakes, how many more do I need to have 10 cupcakes?

Student: "One."

Teacher: "How do you know?"

Student: "Because if you had one more, you could make a ten on the ten frame."

9



K.OA.5
Fluently add and subtract within 5.

"The word *fluent* is used in the Standards to mean "fast and accurate." Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., "adding 0 yields the same number"), and knowing some answers *from the use of strategies*. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students." (*Progressions for the CCSSM (Draft): K, Cardinality; K-5, Operations and Algebraic Thinking, May 2011, p. 18*)

(*continued on next page*)

K.OA.5 (*cont'd*) ***<u>Fluently</u>*** add and subtract within 5.	This Standard is meant to serve as culmination of students' work with the concepts and understandings described in **K.CC.4c**, **K.OA.2**, and **K.OA.4**. Rather than teaching "number facts" as separate items to be memorized, the goal is for students to work with combinations up to 5 with different addition/subtraction problem types, manipulatives, and models throughout the year. Over time, they internalize that (for example) whenever you put 2 things and 3 things together, you get 5 things. Students develop fluency by understanding and internalizing the relationships that exist between and among numbers. Oftentimes, when children think of each "fact" as an individual item that does not relate to any other "fact," they are attempting to memorize separate bits of information that can be easily forgotten. Instead, in order to fluently add and subtract, children must first be able to see sub-parts within a number (**K.CC.4c**, **K.OA.4**). ***<u>TEACHER NOTE</u>***: Research has shown that traditional flash cards or timed tests have not been proven as effective instructional strategies for developing fluency. Rather, numerous experiences with breaking apart actual sets of objects and developing relationships between numbers help children internalize parts of number and develop efficient strategies for fact retrieval.* * Sources include Burns (2000) *About Teaching Mathematics*; Fosnot & Dolk (2001) *Young Mathematicians at Work;* Richardson (2002) *Assessing Math Concepts*; Van de Walle & Lovin (2006) *Teaching student centered mathematics K-3*.

Number and Operations in Base Ten

Cluster

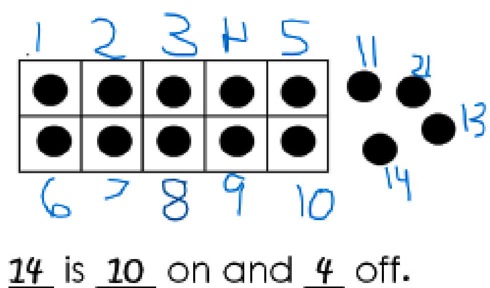
Work with numbers 11-19 to gain foundations for place value.

Vocabulary: tens, ones, place value

K.NBT.1
Compose and decompose numbers from 11 to 19 into tens ones and some further ones to understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8).

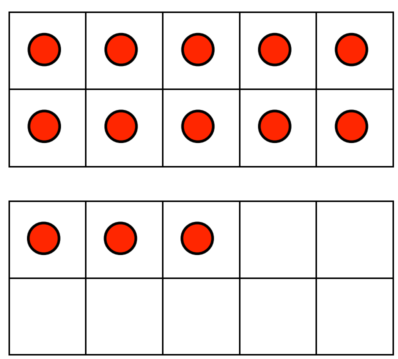
Students continue their work in composing and decomposing numbers (**K.CC.4c**, **K.OA.3**) and begin exploring concepts of our Base 10 place value system. This Standard lays the foundation for further work with place value in Grade 1 in which students “understand that the two digits of a two-digit number represent amounts of tens and ones” (**1.NBT.2**).

Ten Frames, Double Ten Frames, and Rekenreks can be powerful models for exploring these relationships. Students can use single Ten Frames to describe 11 – 19 as some “on the ten frame” and some “off of the ten frame.” For example,



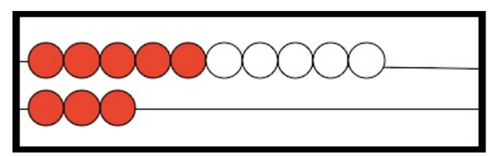
ALL	On	Off
14	10	4

Working with Double Ten Frames or Rekenreks, students can still explore relationships between numbers:



Double Ten Frames

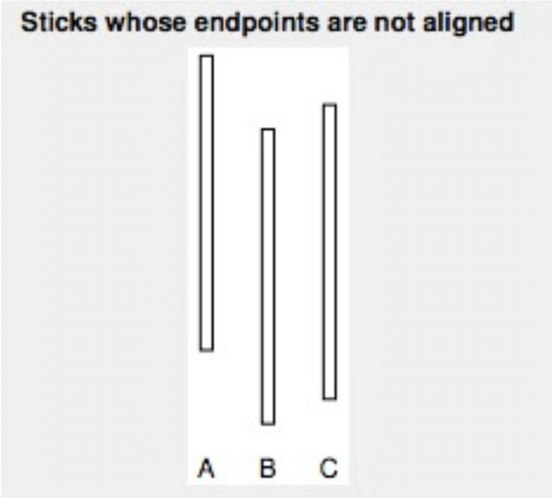
13 = 10 + 3



Rekenrek

13 = 10 + 3

Measurement and Data

Cluster	
Describe and compare measurable attributes.	
Vocabulary: length, long, short, longer, taller, shorter, weight, heavy, light, more, less	
K.MD.1 Describe measureable attributes of objects, such as length or weight. Describe several measureable attributes of a single object.	Young children often use words such as "more" or "bigger" to describe the difference between objects, whether that difference is height, weight, length, space, etc. Teachers can help students develop measurement vocabulary by asking questions such as, "How is it more?" or "How is it bigger?" Teachers should look for opportunities (ex: student conversations, pictures/characters in stories) to help students discuss and explore terms such as big/bigger, small/smaller, long/longer, short/shorter, tall/taller, heavy/heavier, light/lighter, etc.
K.MD.2 Directly compare two objects with a measureable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	A common misconception for young students is to look at where/how the measurement two objects "end up," without considering where/how the measurement began. For example, in the picture below, some students will identify stick A as the "longest" or "tallest" because it sticks out the furthest from the other two. Other students may identify stick B as the longest because it ends up further down on the page than the other two.  (*Progressions for the CCSSM (Draft): K-5, Geometric Measurement, June 2012, p. 6*) Teachers can help children safely confront these misconceptions by orchestrating exaggerated classroom tasks/discussions. For example, the teacher can have one child (safely) stand on a chair and then claim that he/she is "taller than" another child. Students will often claim that this "isn't fair" or "is cheating." The teacher can than ask what about the comparison "isn't fair" and how they can fix it. This can lead to a discussion of the importance of starting at the same place when measuring the height or length of an object.

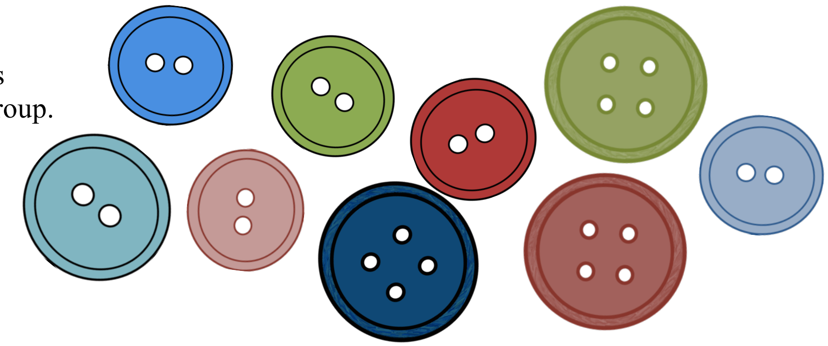
Cluster
Classify objects and count the number of objects in each category.
Vocabulary: sort, how many, most, fewest

K.MD.3
Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]

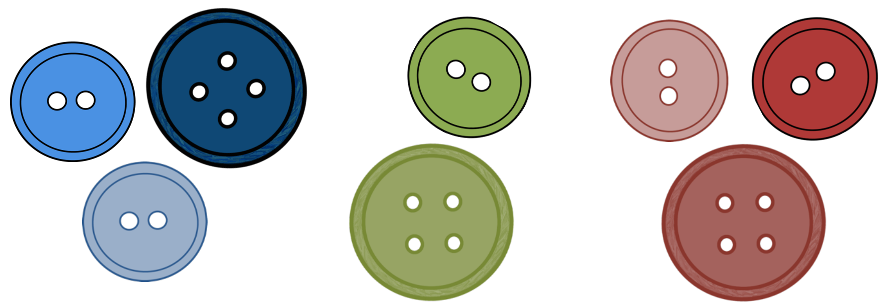
[3] Limit category counts to be less than or equal to 10.

The concepts of one-to-one correspondence and cardinality (**K.CC.4**) are clearly important in this Standard. Students may be provided with sets of objects and asked to sort and count them based on their features. It can be very powerful for students to see that the same set of objects can be sorted in more than one way.

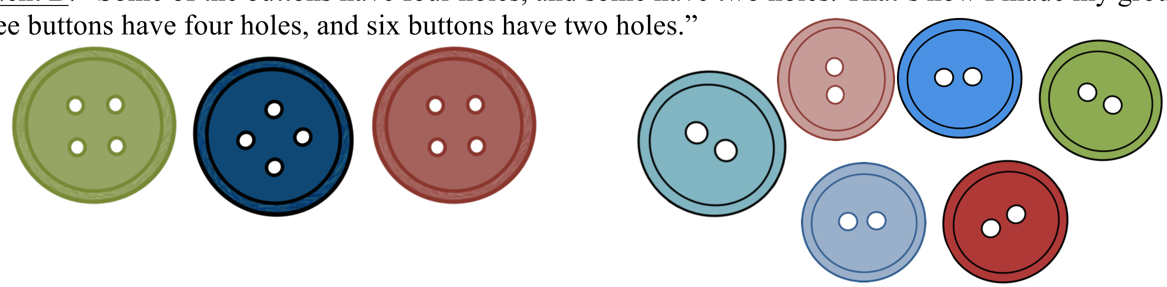
Example:
Let's sort these buttons into groups and count how many are in each group.



Student A: "I used colors for my groups. There are three blues, two greens, and three pinks."



Student B: "Some of the buttons have four holes, and some have two holes. That's how I made my groups. Three buttons have four holes, and six buttons have two holes."



Geometry

Cluster

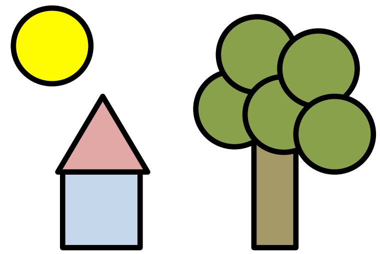
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Vocabulary: square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere, flat, solid, corner/vertex, side, edge, face, above, below, beside, in front of, behind, next to, same, different

K.G.1
Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*

The purpose of this Standard is to allow students to build on their knowledge of familiar objects from everyday life (ex: "ball," "square box," "can") and to make connections with mathematical vocabulary (ex: "sphere," "cube," "cylinder"). In addition, this Standard builds on the State Pre-K standards by allowing students to connect math vocabulary to positional words.

This Standard offers many opportunities for creativity. For example, the teacher might provide students with shape cut-outs and a glue stick and ask them to make a picture using the shapes:



The teacher can use the pictures to ask questions of the students which helps them learn how to describe their pictures using appropriate terminology:

<u>Teacher</u>: What shapes make the house?

<u>Student</u>: A triangle and a square.

<u>Teacher</u>: Is the triangle above the square or below the square?

<u>Student</u>: Above the square.

<u>Teacher</u>: How did you make the tree? What shapes did you use?

<u>Student</u>: I put circles on top of a rectangle.

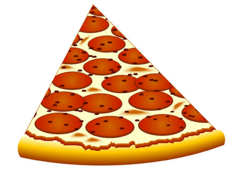
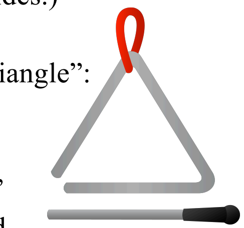
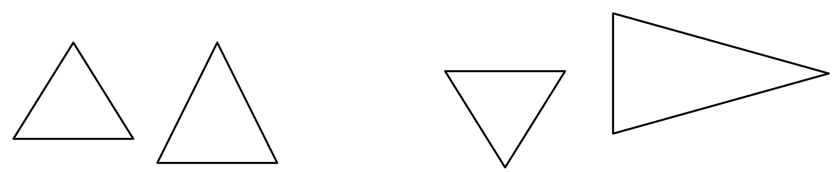
<u>Teacher</u>: How can we describe where the tree is? Is it next to the house? Is it under the house?

<u>Student</u>: Next to the house.

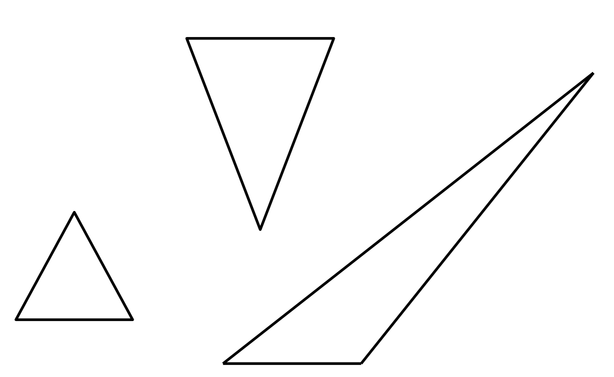
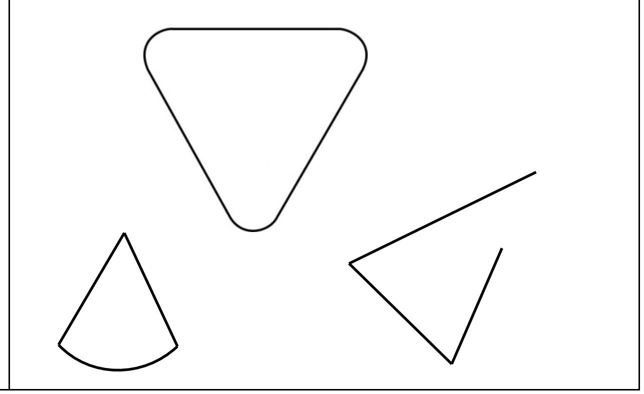
<u>Teacher</u>: If the tree were under the house, where would it be?

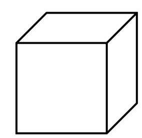
<u>Student</u> <points under the house>: Under the house is here.

(*continued on next page*)

K.G.1 (*cont'd*) Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*	 ***TEACHER NOTE***: It is important that teachers be thoughtful in their selection of "real world" examples of shapes. A typical pizza slice is not a good example of a triangle because it has a curved side; triangles have three *straight* sides. Students may initially make similar claims because, in general, "It looks like a triangle." A way to help them address this misunderstanding in a positive way is to clarify, "It's *almost* a triangle. What could we change to make the pizza slice a triangle?" (Make it have three straight sides.)  There is also some natural confusion at this age about the musical instrument called a "triangle": The instrument is called a "triangle" because it obviously resembles a triangle. *However*, you will notice that there is a gap between two of the three sides. This does not fit the mathematical definition of a triangle: Triangles have three straight sides and are "closed" (all sides touching with no gaps between them). But this "conflict" can be easily resolved with young children with similar conversations to the pizza slice above: "Musicians call this instrument a triangle because it looks a lot like a triangle. But we know it's not a 'math triangle.' What could we change to make it a math triangle?" (Make sure that all sides touch and that the sides are straight, not curved.)
K.G.2 Correctly name shapes regardless of their orientations or overall size. (*continued on next page*)	Students typically enter Kindergarten with a general knowledge of shapes. However, their mental picture of a shape is often limited to an "ideal" version of the shape. For example, many young students will identify an equilateral triangle or isosceles triangle as a triangle because "it looks like" the triangles they have seen. However, they may also claim that triangles with different orientations or elongated sides are *not* triangles because "they don't look like triangles":  Triangles "Not Triangles" Young Children's Initial Claims, Based on What the Shapes "Look Like" Classroom conversations can help resolve these misconceptions. Teachers can facilitate age-appropriate discussions about "What 'makes' this shape a triangle?" and guide students through attention to specific features (ex: three straight sides that touch) versus general, yet ultimately inaccurate ideas of "pointy" or "flat on the bottom."

<table>
<tr>
<td>K.G.2 (cont'd)
Correctly name shapes regardless of their orientations or overall size.</td>
<td>Research also shows that presenting young students with examples and non-examples (see below) and facilitating discussions of "What do you notice?... What is 'the same' about the triangles? Why are the other shapes not with the triangles?" can help them learn how to pay attention to defining features of shapes (ex: number of sides, straight sides, etc.) rather than non-defining features (ex: orientation, size, color).

<table>
<tr><th>These are triangles</th><th>These are not triangles</th></tr>
<tr><td></td><td></td></tr>
</table>
</td>
</tr>
<tr>
<td>K.G.3.
Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").</td>
<td>This intent of this Standard is to introduce students to the concept of two-dimensional and three-dimensional shapes in a very informal way. In general, students should be able to describe cubes, cones, spheres, and cylinders as "solid" shapes and hexagons, squares, rectangles, circles, and triangles as "flat" shapes.

<u>TEACHER NOTE</u>: It is important that teachers be thoughtful about the language they use to describe shapes. Some educational resources may show a picture of a cube and use it to reference a square. <u>But the cube is not an example of a square</u>. The flat face of a cube is a square.


Similar caution should be used with using a "ball" as an example of a circle – A ball is three-dimensional (a sphere), not two-dimensional (a circle).


Some may question if this level of detail is necessary for children at this age.
Research indicates that by the age of six, children may already have established mental images and general definitions for what a shape is, based on their exposure to shapes (Hannibal, 1999). So, it is important that teachers make careful choices about shapes and shape-discussions in their classrooms.</td>
</tr>
</table>

Cluster

Analyze, compare, create, and compose shapes.

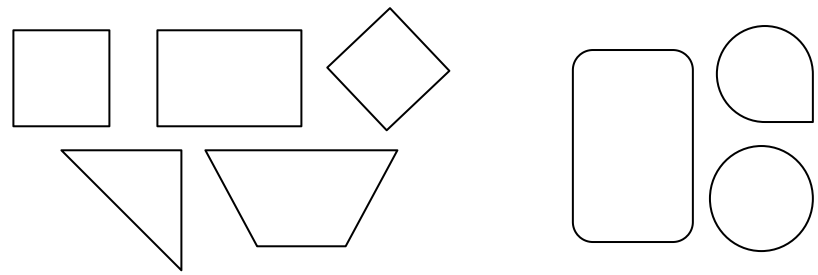
Vocabulary: two-dimensional (flat), three-dimensional (solid), side, corner/vertex, corners/vertices, edge, face, same, different

K.G.4
Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).

(*continued on next page*)

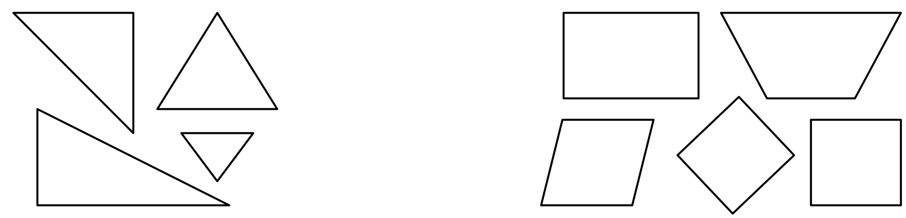
In **K.G.3**, students explore the differences between two-dimensional and three-dimensional shapes.
In **K.G.4**, students look within those categories to compare and contrast shapes.

For example, students might sort a set of two-dimensional shapes into two groups:



“Shapes with all straight sides” “Shapes with curves”

Or students might sort a set of two-dimensional shapes based on the number of straight sides they have:



“These shapes have three straight sides.” “These shapes have four straight sides.”

Students might also count the number of shapes in each group, connecting to Standard **K.MD.3**.

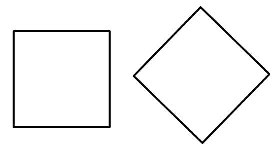
TEACHER NOTE: In the U.S., the term “trapezoid” may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. With this definition, parallelograms, rectangles, squares, and rhombi fit that definition and can thus be considered as *types of trapezoids*. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, parallelograms and their subgroups do not fit the definition and thus are not considered to be types of trapezoids. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012)

<table>
<tr>
<td>K.G.4 (cont'd)
Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).</td>
<td>

<u>TEACHER NOTE</u>: The mathematical attributes of a rectangle *do not include* "having two long sides and two short sides." Those characteristics <u>should not</u> be taught as defining attributes of a rectangle. In its most general terms, a rectangle has four straight sides and four "square corners" (or right angles). The opposite sides of a rectangle are the same length and are parallel. (If you extended the sides, they would never touch.) A square fits all of the characteristics of a rectangle. A square is a *special type of rectangle* in that <u>all</u> sides of a square are the same length.

A developmentally appropriate understanding of the relationship between squares and rectangles might be that "squares are part of the rectangle family."

It is also important to note that orientation of a figure does not change the figure itself. Given the shapes below, students often refer to the figure on the left as a square and the figure on the right as a diamond. <u>Both figures are squares</u>; the square on the right has just been rotated. (This connects back to **K.G.2**.)



Unfortunately, many "educational" materials refer to a rhombus, or even a rotated square, as a "diamond." "<u>Diamond" is not a geometric term and should not be used to describe shapes</u>.

The following articles (written for teachers) offer helpful advice for discussing shapes with young students:

- Hannibal, M. A. (1999, February). Young children's developing understanding of geometric shapes. *Teaching Children Mathematics*, 353-357.
- Roberts, S. K. (2007, December). Watch what you say. *Teaching Children Mathematics*, 296-301.

</td>
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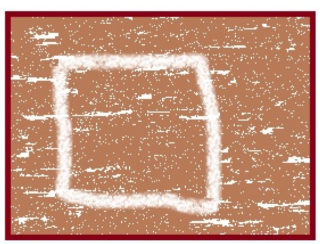
<tr>
<td>K.G.5
Model objects in the world by drawing two-dimensional shapes and building three-dimensional shapes.</td>
<td>

This Standard is intended to provide students with tactile, hands-on experiences with constructing shapes while their fine motor skills are developing. Just as students may "draw" letters in sand, they might also draw shapes. Teachers might also give students mats with shape outlines and ask students to use play dough/clay to make the outlines and name the shapes:




</td>
</tr>
</table>

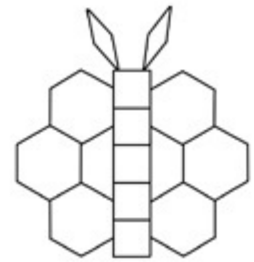
K.G.6
Compose simple shapes to form larger shapes.

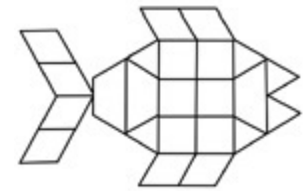
For example, "Can you join these two triangles with full sides touching to make a rectangle?"

This Standard is intended to build off of **K.G.2** – shapes can be turned (flipped, rotated, etc.) in different orientations and joined with other shapes to make new shapes. The concept of composing/decomposing shapes is very important to students' future work with rectangular arrays and area models in later grades.

<u>Examples of relevant tasks include</u>

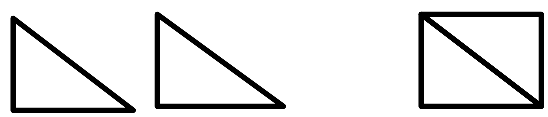
(1) Using pattern blocks to "tile" or "fill" other shapes:



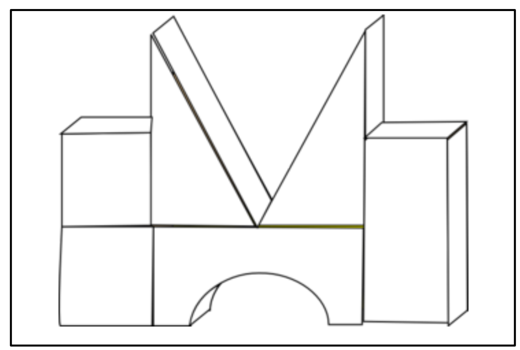


<u>TEACHER NOTE</u>: Teachers should take advantage of these "simple" tasks to discuss mathematical vocabulary and to work on counting skills. Example: "What shapes make the antennae of the butterfly?" (rhombuses or rhombi) "How many rhombi did you use to make the fish's tail?" (four)

(2) Specific requests: "Can you join these two triangles with full sides touching to make a rectangle?"



"Look at the picture of the castle. Use the building blocks to make a castle like the one in the picture:



(3) "Open-ended": "Let's use the pattern blocks to make a face for the scarecrow."

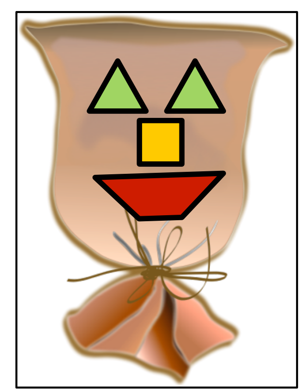


Table 1: Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ? (K)	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5 (1st)	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5 One-Step Problem (2nd)
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ? (K)	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3 (1st)	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3 One-Step Problem (2nd)
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put Together/Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ? (K)	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5 or 5 – 3 = ? (1st)	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2 (K)
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (1st)	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? One-Step Problem (1st)	(Version with “more”): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? 5 – 3 = ? or ? + 3 = 5 One-Step Problem (2nd)
Compare	(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5 or 5 – 2 = ? (1st)	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ? or 3 + 2 = ? One-Step Problem (2nd)	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5 One-Step Problem (1st)

K: Problem types to be mastered by the end of the Kindergarten year. **1st**: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types. **2nd**: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

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